Polyhedral Modeling

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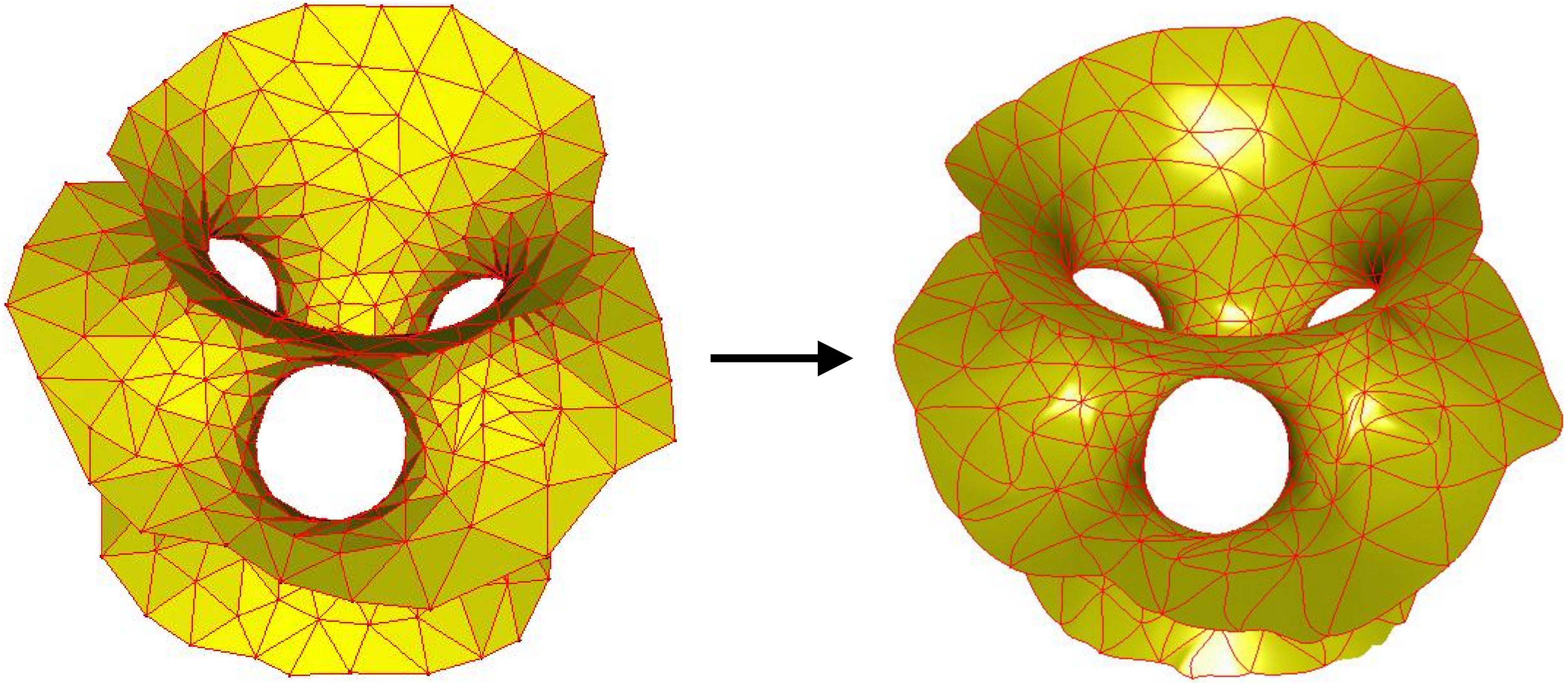
Modeling smooth polyhedra



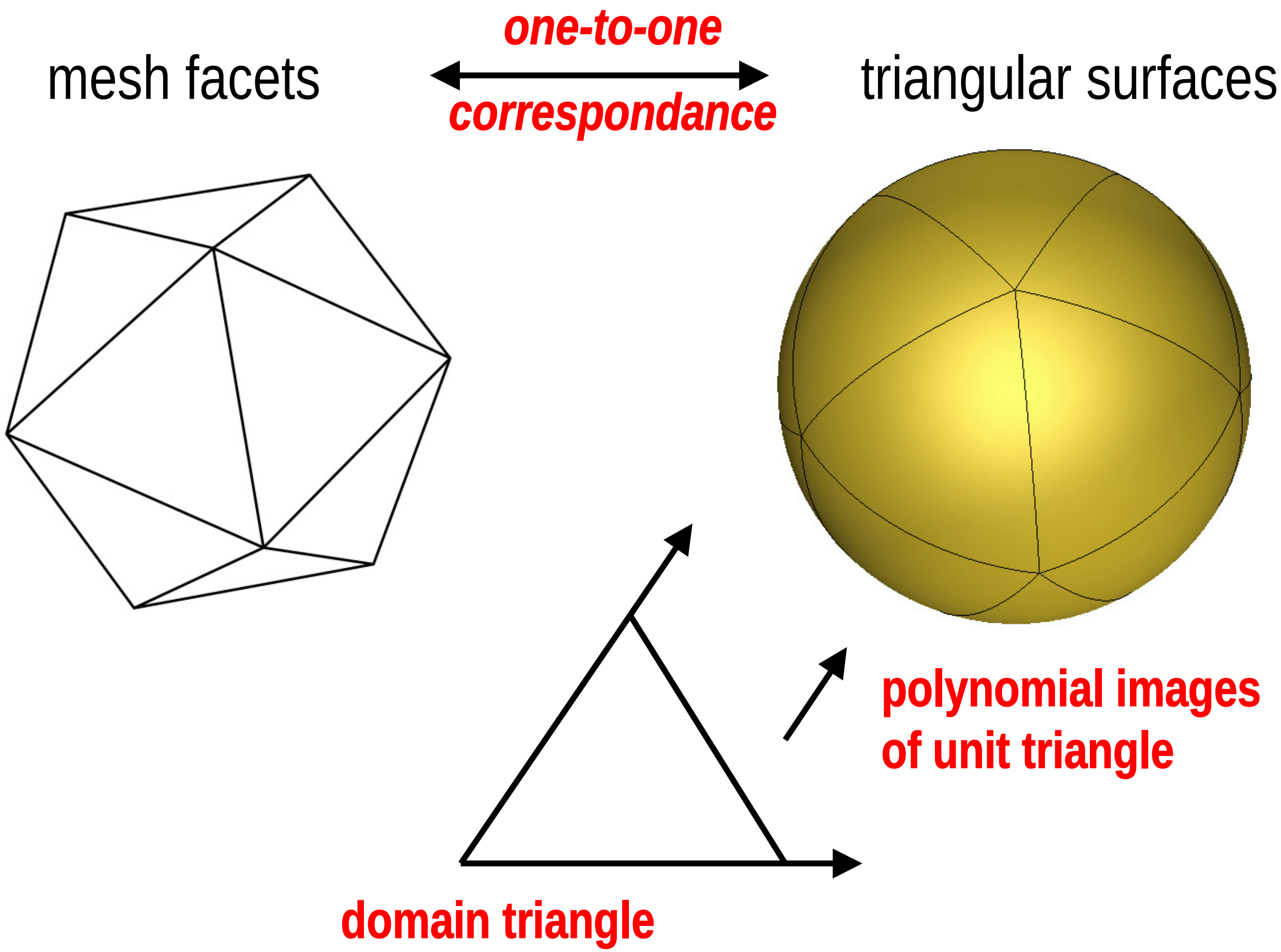
Polyhedral mesh

- triangular faces
- arbitrary topology (2d manifold)

Smooth surface

- mesh interpolation
- parametric, polynomial
- local support

Parametric surfaces of arbitrary topology



Outline

the 4-split method – basic idea

G1 continuity

Related works

the algorithm

step I. boundary curve network

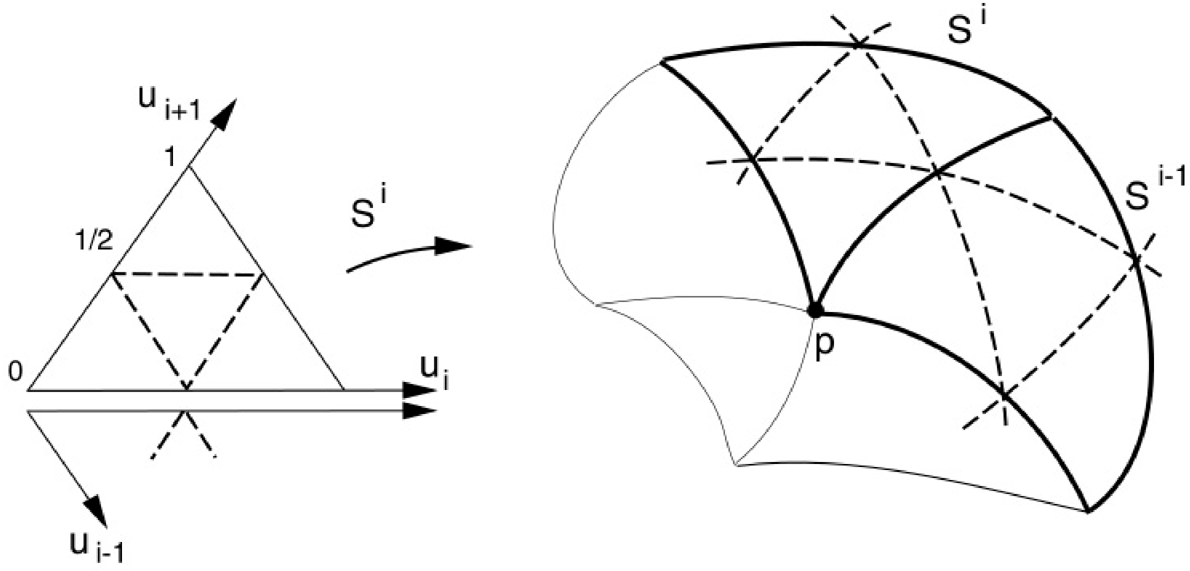
step II. cross boundary tangents

step III. fill-in patches

Results

Future work

domain 4-split



- 4 triangular Bezier patches per ***macro-patch*** S^i
- piecewise boundary curves
- piecewise cross-boundary tangents

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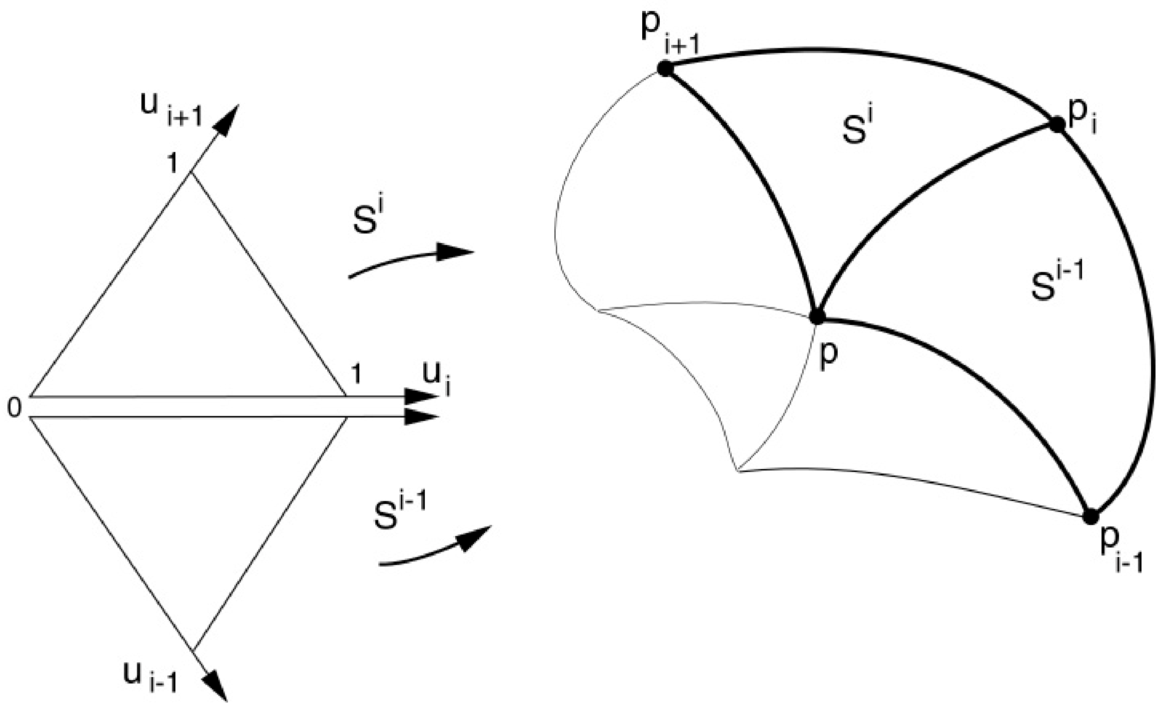
Parametrization

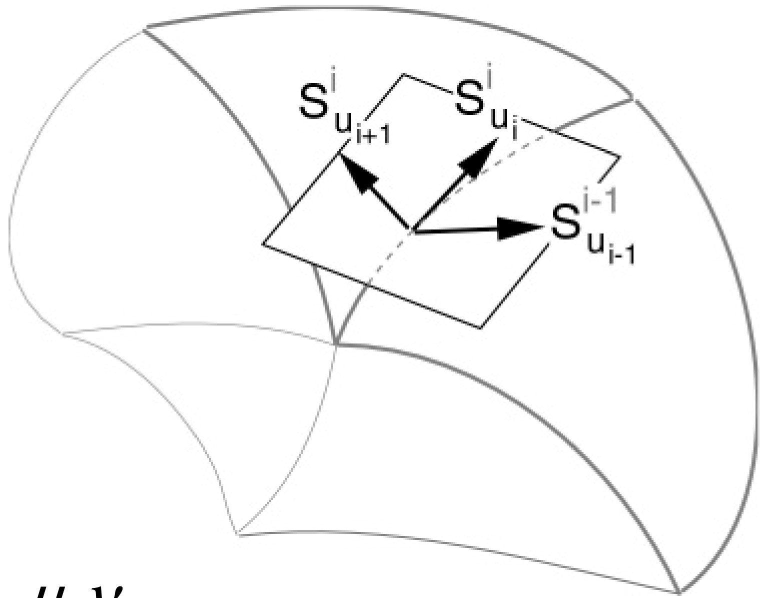


S^i **macro patch** (4 triangular Bézier patches)

p mesh vertex

p_i vertex neighbourhood, i=1, . . . , n

Tangent plane continuity



between adjacent patches :

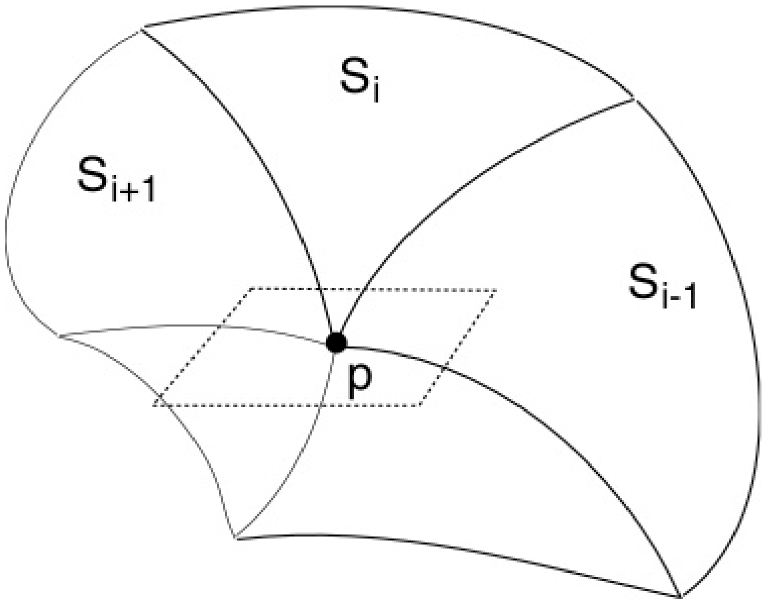
- common boundary of S^{i-1} and S^i
- existence of 3 scalar functions : Φ, μ, ν

$$\Phi\, S^i_{u_i} = \mu\, S^{i-1}_{u_{i-1}} + \nu\, S^i_{u_{i+1}}$$

continuity constraint

we choose : $\mu \equiv \frac{1}{2}, \nu \equiv \frac{1}{2}$

Tangent plane continuity



Vertex of n patches:

at a common vertex **p**, the existence of a well defined tangent plane is not sufficient.

Polynomial patches need to be **twist compatible:**

$$s^i_{u_i u_{i+1}}(0,0) = s^i_{u_{i+1} u_i}(0,0)$$

twist constraint

$$\frac{1}{2} s^{i-1}_{u_i u_{i-1}}(0,0) + \frac{1}{2} s^{i}_{u_i u_{i+1}}(0,0) = \Phi^1 s^i_{u_i}(0,0) + \Phi^0 s^i_{u_i u_i}(0,0)$$

t_i $\quad t_{i+1}$ $\quad r^1_i$ $\quad r^2_i$ $\quad$ **i=0,…,n**

Tangent plane continuity at vertex **p**

matrix notations:

continuity constraint:

$$(P)\; r^1 = 0$$

$$P = \begin{pmatrix} -\Phi^0 & \frac{1}{2} & 0 & \cdots & & \frac{1}{2} \\ \frac{1}{2} & -\Phi^0 & \frac{1}{2} & & & 0 \\ 0 & \ddots & \ddots & \ddots & & \vdots \\ & & \ddots & \ddots & \ddots & \\ & & & \frac{1}{2} & -\Phi^0 & \frac{1}{2} \\ \frac{1}{2} & & & & \frac{1}{2} & -\Phi^0 \end{pmatrix}$$

twist constraint:

$$(T)\; t = \Phi^1 r^1 + \Phi^0 r^2$$

$$T = \begin{pmatrix} \frac{1}{2} & & & & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & & & 0 \\ & \ddots & \ddots & & \\ & & \ddots & \ddots & \\ 0 & & & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

twist compatibility problem: **T** may be singular !!!

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Related works

- Clough-Tocher domain splitting methods (Farin'82, Piper'87, Shirman/Sequin'87)
- Convex combination schemes (Gregory'86, Hagen'86, Nielson'87)
- Boundary curve schemes (Peters'91, Loop'94)
- Algebraic methods (Bajaj'92)
- 4-split method (HB'00)

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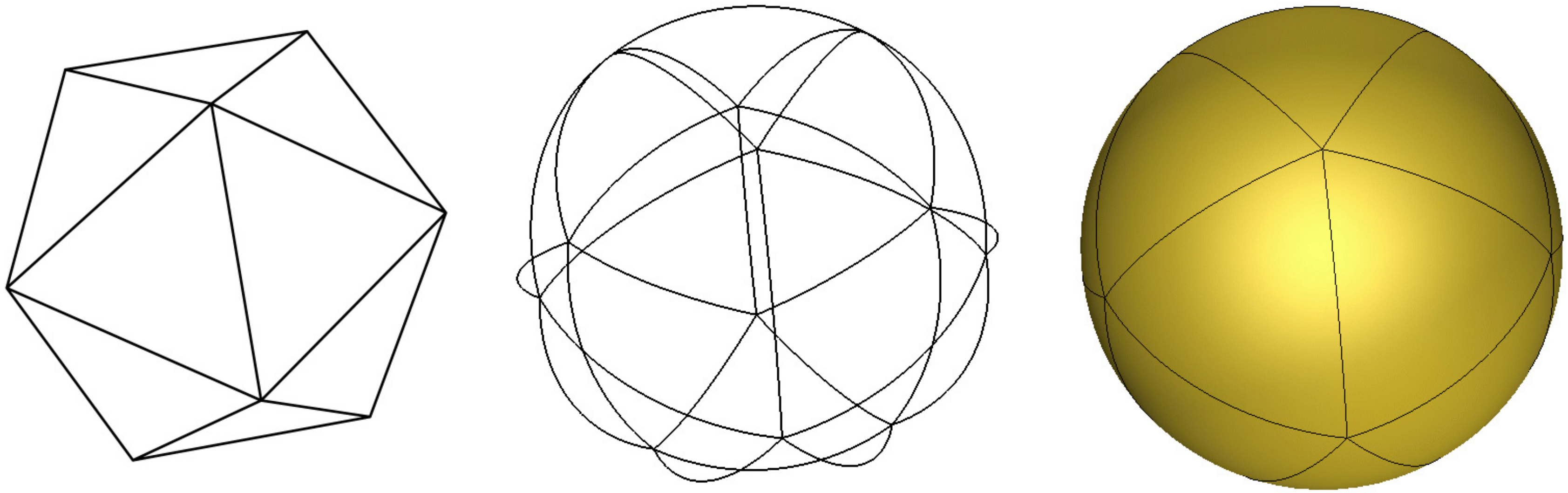

3 steps:

I. boundary curve network

II. cross boundary tangents

III. fill-in patches

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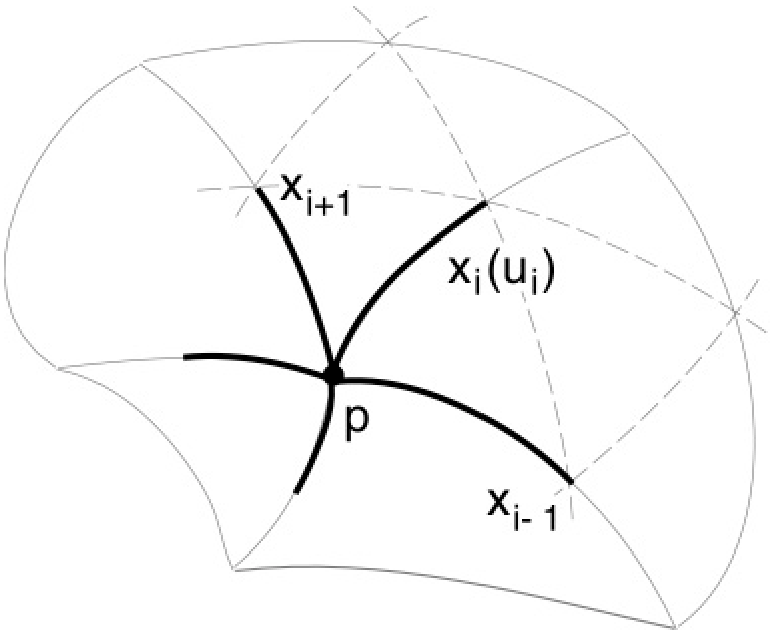
I. boundary curves

Local curve network construction :



- local first and second derivatives at **p**
- interpolation of mesh vertex:

  $\mathbf{x_i(0) = p}$

- according to continutiy and twist constraints

  $\mathbf{r^1 = [\, x'_1(0), \ldots, x'_n(0)\,]}$ lies in **ker (P)** and **Im (T)**

  $\mathbf{r^2 = [\, x''_1(0), \ldots, x''_n(0)\,]}$ lies in **Im (T)**

- C1-join of both curve pieces along edge

=> **Piecewise cubic curves**

- **null space of P**

Ker (P) = span{$\mathbf{k}_1$, $\mathbf{k}_2$}

$\mathbf{k}_1$, $\mathbf{k}_2$ $\in$ **Im (T)**

$$k_1 = \begin{bmatrix} 1 \\ \vdots \\ \cos\left(\frac{2i\pi}{n}\right) \\ \vdots \\ \cos\left(\frac{2(n-1)\pi}{n}\right) \end{bmatrix}, \quad k_2 = \begin{bmatrix} 0 \\ \vdots \\ \sin\left(\frac{2i\pi}{n}\right) \\ \vdots \\ \sin\left(\frac{2(n-1)\pi}{n}\right) \end{bmatrix}$$

- **image space of T**

$$\text{Rank}(T) = \begin{cases} n & \text{if } n \text{ is odd} \\ n-1 & \text{if } n \text{ is even} \end{cases}$$

$$\mathbf{Im}(\mathbf{T}) = \{ \mathbf{l}_1, \ldots, \mathbf{l}_{n \text{ or } n-1} \}$$

boundary curves (cont.)

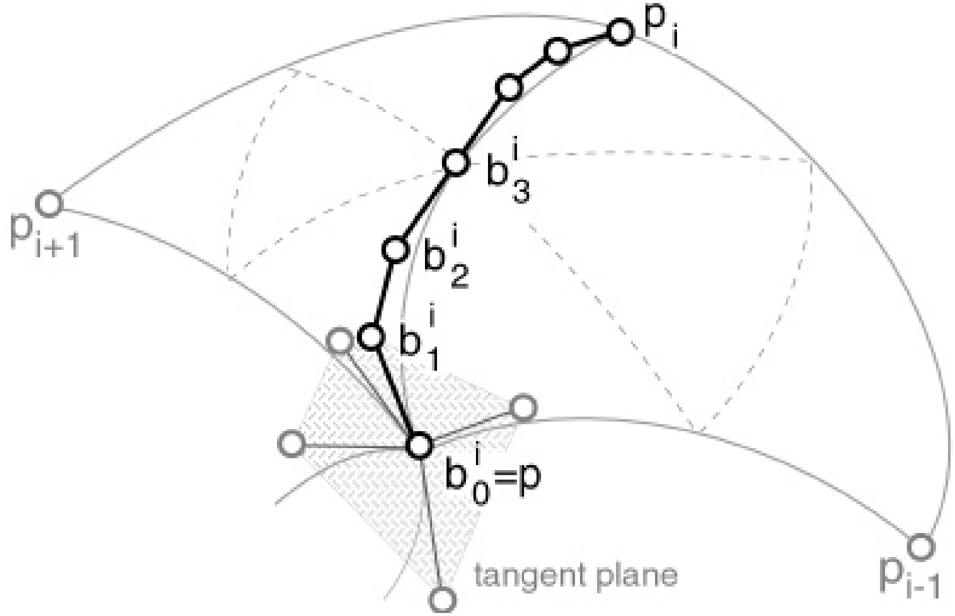
Bézier control points:

$$b_0 = p$$

$$b_1 = \left[k_1 \right]_{n \times 1} \left[u_1 \right]_{1 \times 3} + \left[k_2 \right]_{n \times 1} \left[u_2 \right]_{1 \times 3}$$

$$b_2 = l_i v_i$$

$$b_3 = \frac{1}{2}\left(b_2^L + b_2^R \right)$$



many degrees of freedom:

2 vectors for the *n* first derivatives

n or **n-1** vectors for the *n* second derivatives

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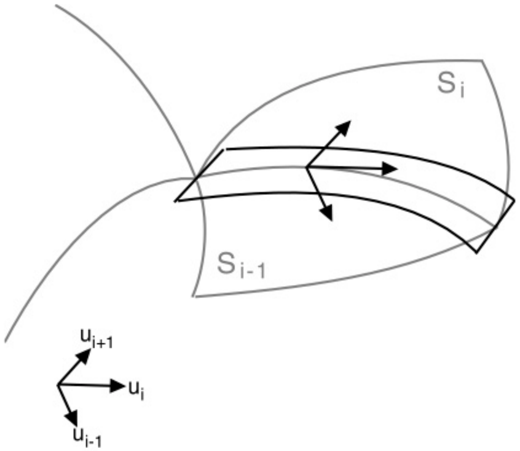
II. cross boundary tangents



$$S^{i}_{u_{i+1}} = \Phi_i \, S^{i}_{u_i} + W_i$$

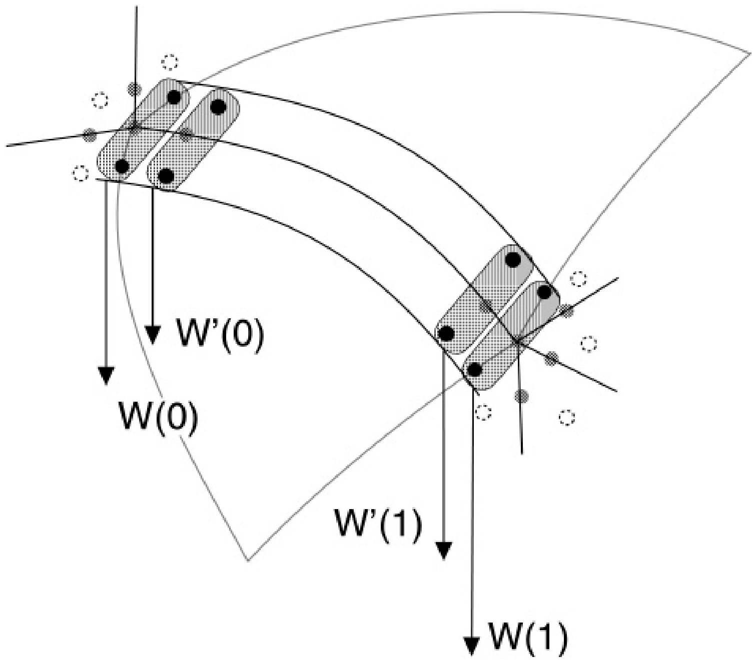
$$S^{i-1}_{u_{i-1}} = \Phi_i \, S^{i}_{u_i} - W_i$$

d⁰ 5 patch

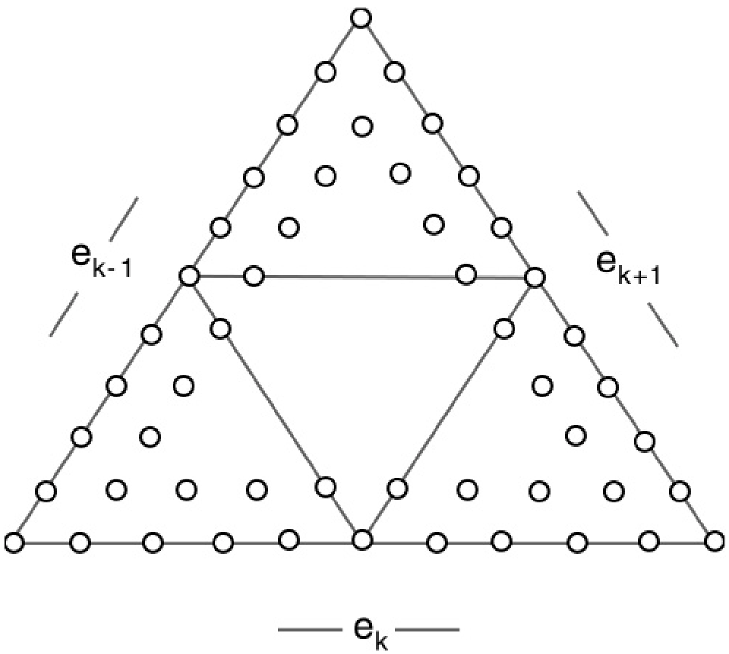
d⁰ 1 **d⁰ 2 => d⁰ 3**

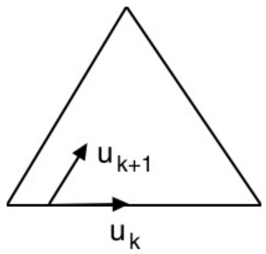
$$\frac{1}{2} S^{i-1}_{u_{i-1}} + \frac{1}{2} S^{i}_{u_{i+1}} = \Phi_i S^{i}_{u_i}$$

conditions on W_i :



cross boundary tangents (cont.)





boundary control points:

degree elevation: d^0 3 → d^0 5

first row of control points :

degree elevation: d^0 3 → d^0 4

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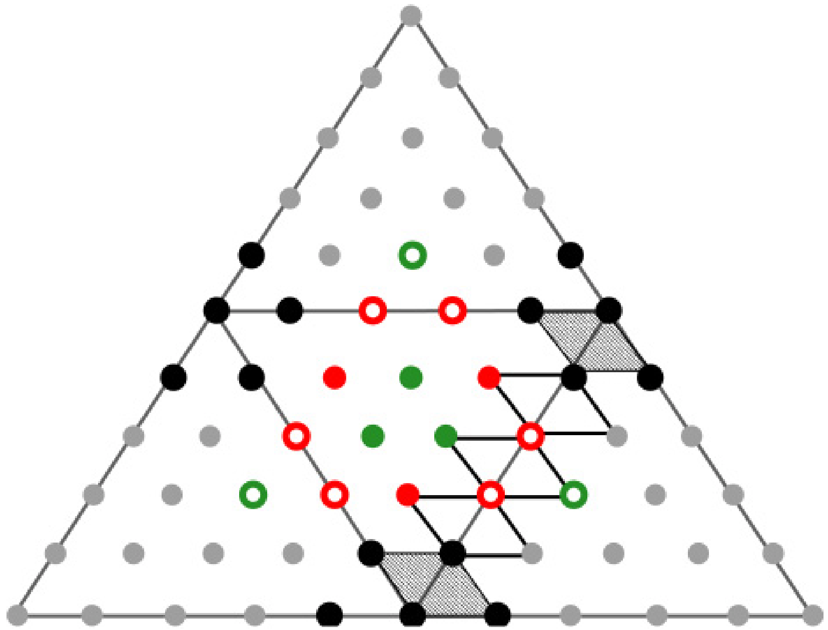
III. fill-in patches



- edge mid-points are C1-continuous
- two triang. surfaces are C1 at common boundary
  the three rows of cp form parallelograms

Make inner edges C1 continuous:

(1) choose ● => determines ○

(2) choose ● => determines ○

- 6 degrees of freedom

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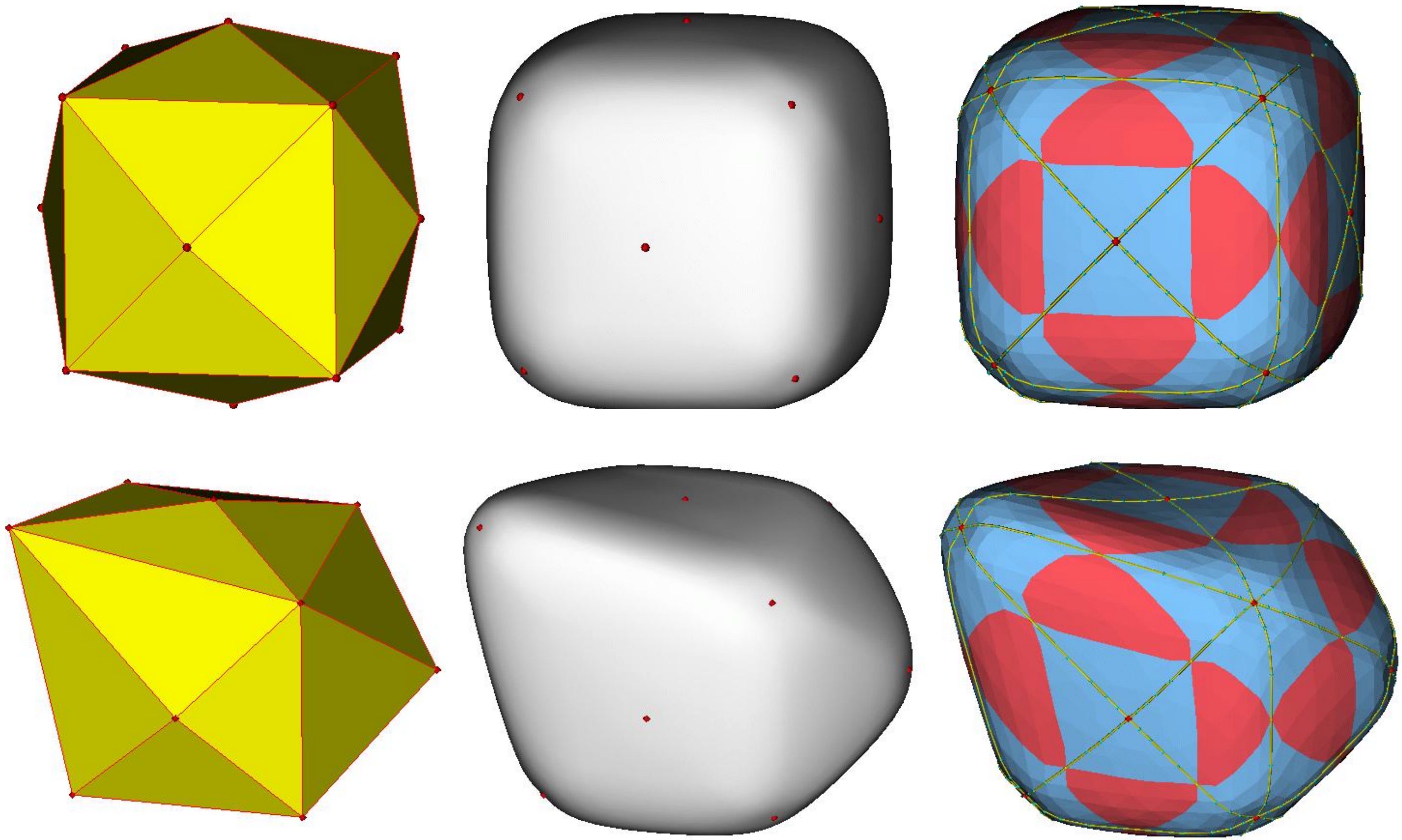

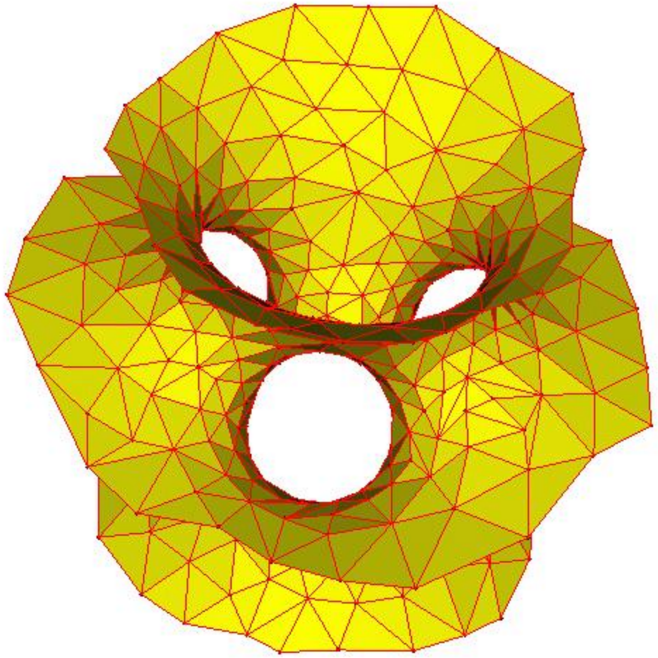

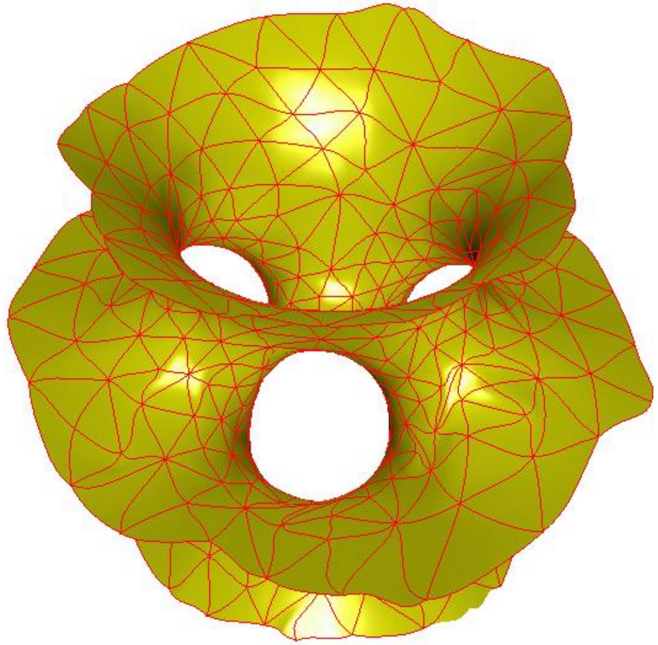

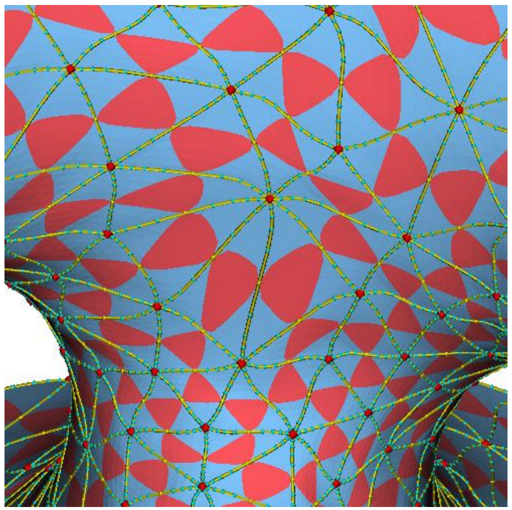

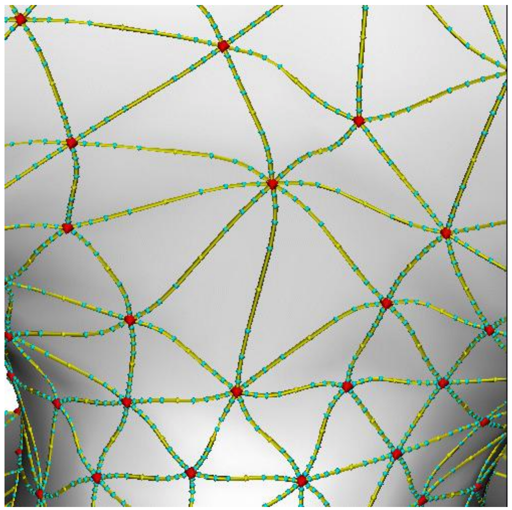

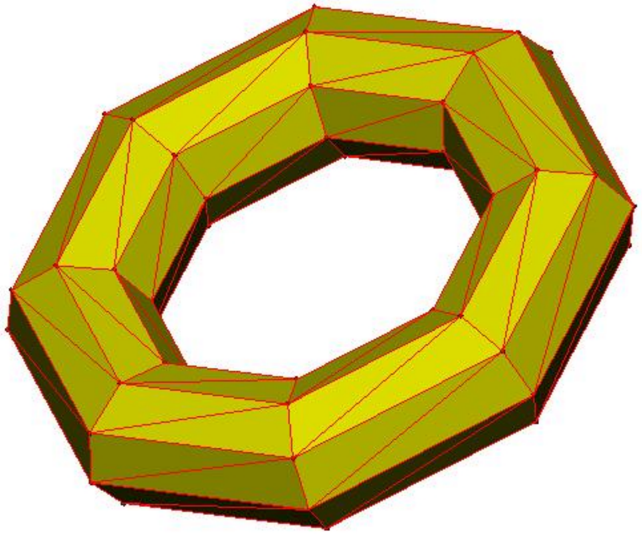

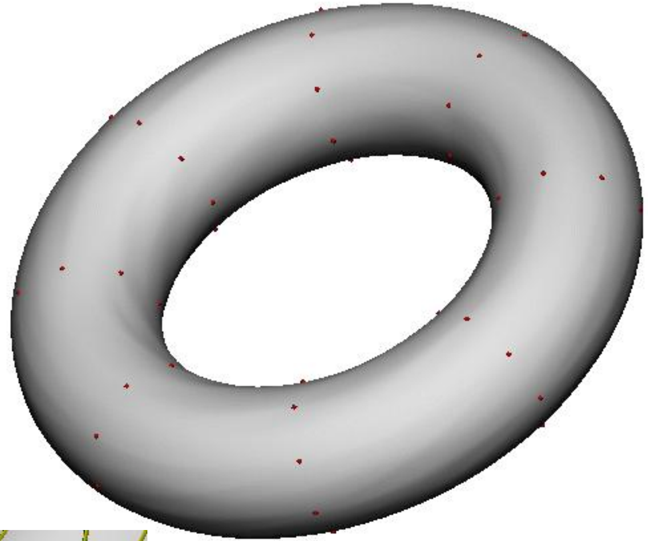

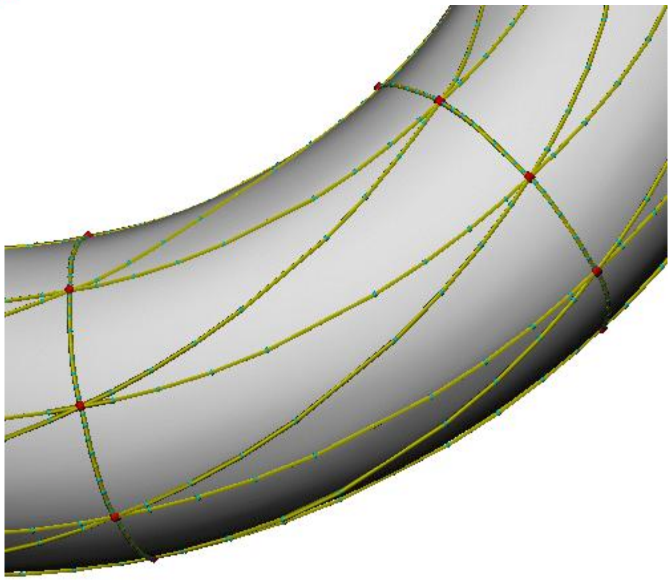

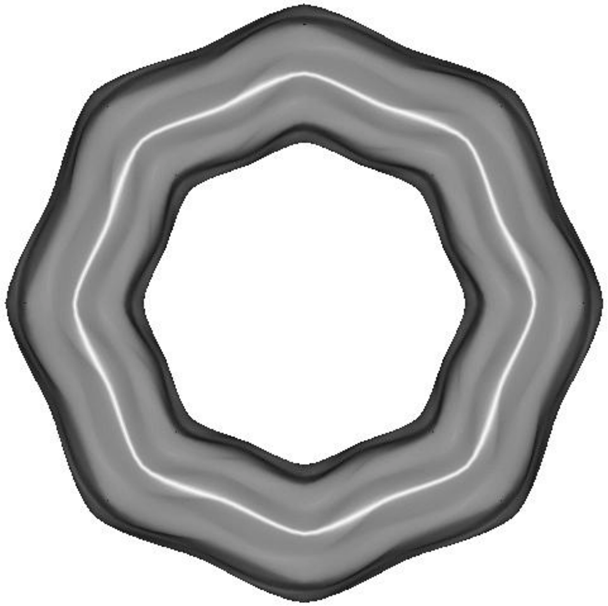

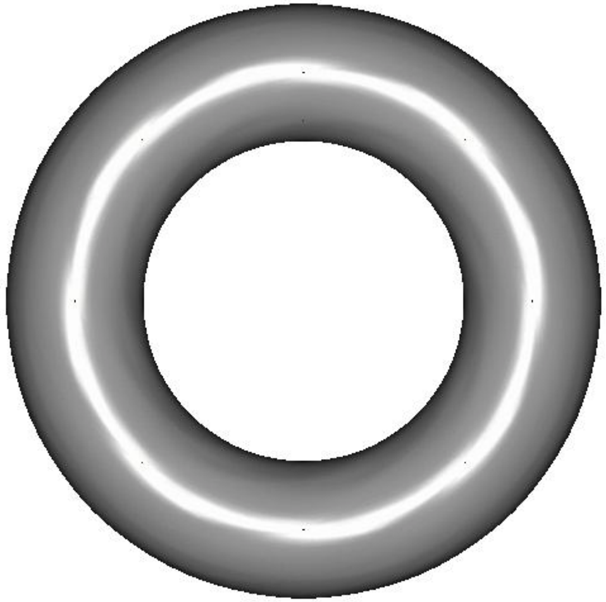

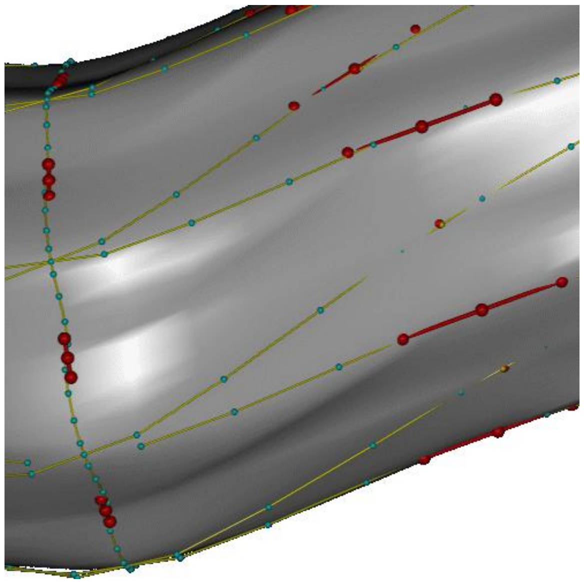

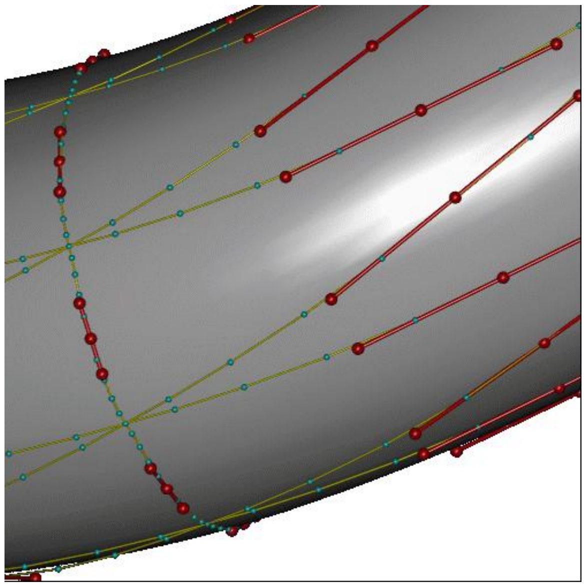

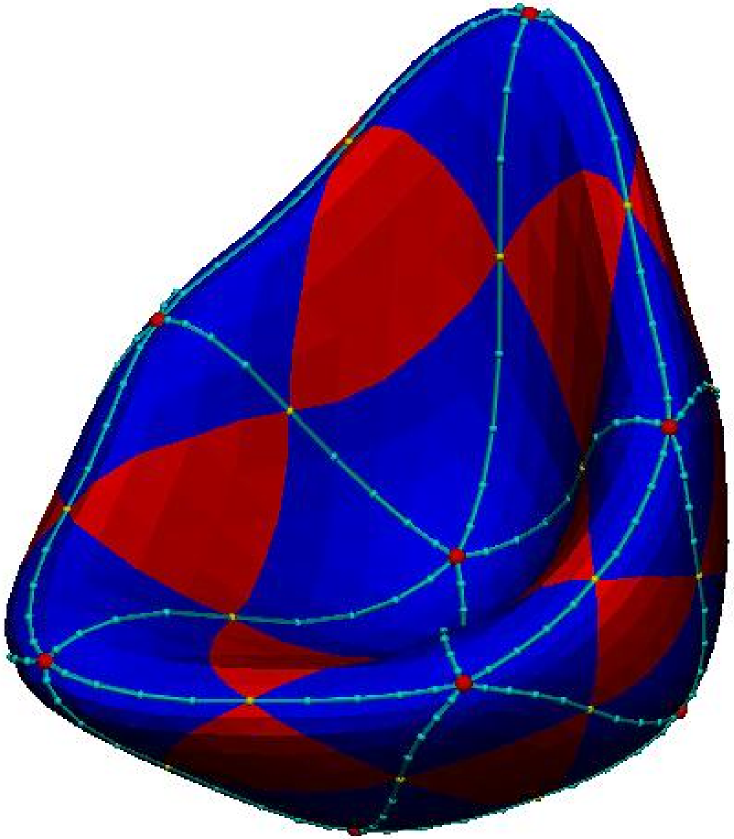

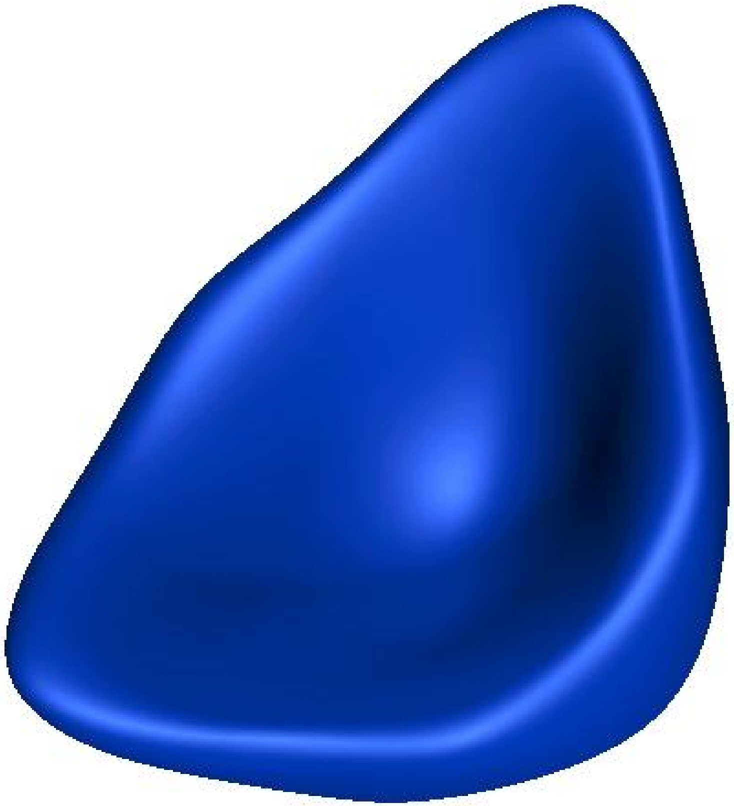

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- 4-split method – basic idea
- G1 continuity
- Related works
- the algorithm
- step I.   boundary curve network
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Conclusion and future work

conclusion :

- arbitrary topology (2d manifold)
- 4-split of domain triangle
- local scheme
- closed form, explicit representation
- 4 quintic patches per mesh face.

future work :

- approximate iso-surfaces
- optimal choice of free parameters
- arbitrary choice of first derivatives
- **multiresolution** : the interpolant is refinable